

1. If  $f(x) = xe^{x(1-x)}$ ,  $x \in \mathbb{R}$ , then  $f(x)$  is
  - (a) Decreasing on  $(-\frac{1}{2}, 1]$
  - (b) Decreasing on  $(\frac{1}{2}, 1)$
  - (c) Decreasing on  $(\frac{1}{2}, 1]$
  - (d) None of the above
2.  $\int (\frac{1}{x} \tan(\frac{1}{x}))^2 dx$  is equal to
  - (a)  $x - \tan(x) + C$
  - (b)  $\frac{1}{x} + \tan(\frac{1}{x}) + C$
  - (c)  $\frac{1}{x} - \tan(\frac{1}{x}) + C$
  - (d) None of the above
3. If  $\log_{10}(x^3 + y^3) - \log_{10}(x^2 + y^2 - xy) \leq 2$ , then the maximum value of  $xy$  for all  $x \geq 0, y > 0$ , is
  - (a) 2500
  - (b) 3000
  - (c) 1200
  - (d) None of the above
4. If  $f(x) = ax + b$  and  $f^{-1}(x) = bx + a$ , with  $a, b, x \in \mathbb{R}$ , then what is the value of  $a + b$ ?
  - (a) -2
  - (b) -1
  - (c) 0
  - (d) 1
5. If,  $\ln(a + c), \ln(c - a), \ln(a - 2b + c)$  are in Arithmetic Progression, then
  - (a)  $a, b, c$  are in Arithmetic Progression
  - (b)  $a, b, c$  are in Geometric Progression
  - (c)  $a, b, c$  are in Harmonic Progression
  - (d) None of the above

6. Let  $x > 0$  and  $\log_2 x + \log_2 \sqrt{x} + \log_2 \sqrt[4]{x} + \log_2 \sqrt[8]{x} + \dots = 4$ . Then  $x$  is equal to
- 2
  - 3
  - 4
  - None of the above
7. A fair coin is tossed  $n$  times. If the probability that head occurs 6 times is equal to the probability that head occurs 8 times, then the value of  $n$  is
- 14
  - 16
  - 24
  - None of the above
8. If  $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$  and  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , then  $I + 2A + 3A^2 + \dots$  is equal to
- $\begin{bmatrix} 4 & 1 \\ -4 & 0 \end{bmatrix}$
  - $\begin{bmatrix} 3 & 1 \\ -4 & 1 \end{bmatrix}$
  - $\begin{bmatrix} 5 & 2 \\ -8 & -3 \end{bmatrix}$
  - $\begin{bmatrix} 5 & 2 \\ -3 & -8 \end{bmatrix}$
9.  $\lim_{x \rightarrow \frac{\pi}{4}} [\tan(x)]^{\tan(2x)}$  is equal to
- 1
  - $e$
  - 1
  - None of the above

10. The sum of all squared numbers between 50 and 500 is
- (a) 3704
  - (b) 3655
  - (c) 4233
  - (d) None of the above
11. Coefficient of  $x^{99}$  in the expansion of  $(x + 1)(x + 3)(x + 5)\dots(x + 199)$  is equal to
- (a) 10250
  - (b) 10000
  - (c) 10500
  - (d) None of the above
12. If  $N = n!$ , where  $n$  is a natural number with  $n > 2$ , then

$$\lim_{N \rightarrow \infty} [\log_2 N]^{-1} + [\log_3 N]^{-1} + [\log_4 N]^{-1} + \dots + [\log_n N]^{-1}$$

is,

- (a) 1
  - (b) 2
  - (c) 3
  - (d) None of the above
13. The final score in a recreational soccer game between Team A and Team B, is 6 goals for A to 3 goals for B. How many possibilities exist for the score at the end of first half?
- (a) 20
  - (b) 24
  - (c) 28
  - (d) None of the above

14. Integers  $a, b, c$  and  $d$ , not necessarily distinct, are chosen independently and at random from 0 to 2007 (both inclusive). What is the probability that  $ad - bc$  is even?

- (a)  $\frac{3}{8}$
- (b)  $\frac{7}{16}$
- (c)  $\frac{9}{16}$
- (d)  $\frac{5}{8}$

15. If the function  $f$  satisfies the relation  $f(x+y) = f(x)f(y)$  for all  $x, y \in \mathbb{N}$ . Further if  $f(1) = 2$  and  $\sum_{k=1}^n f(a+k) = 16(2^n - 1)$ , then value of  $a$ , (where  $a \in \mathbb{N}$ ), is equal to

- (a) 3
- (b) 1
- (c) 2
- (d) 4

16. If for any real number  $y$ ,  $[y]$  is the greatest integer less than or equal to  $y$ , then the value of the integral  $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [2 \sin x] dx$ , is

- (a)  $-\pi$
- (b) 0
- (c)  $-\frac{\pi}{2}$
- (d) None of the above

17. The value of real number  $x$  for which the matrix  $\begin{bmatrix} 6 & 4 & 2x \\ 8 & 2 & -2 \\ 0 & 6 & 8 \end{bmatrix}$  has no inverse is

- (a)  $\frac{12}{11}$
- (b)  $\frac{11}{12}$
- (c) 1
- (d) 0

18. The sum of the infinite series

$$\frac{5}{13} + \frac{55}{13^2} + \frac{555}{13^3} + \dots$$

is equal to

- (a)  $\frac{31}{18}$
  - (b)  $\frac{65}{32}$
  - (c)  $\frac{65}{36}$
  - (d)  $\frac{75}{36}$
19. Water is being poured at the rate of 2 cubic metres per second into a cone which has semi vertical angle of  $45^\circ$ . The rate at which perimeter of water surface changes when the height of water in the cone is 2 metres is
- (a) 2 metres per second
  - (b) 1 metre per second
  - (c) 3 metres per second
  - (d) 4 metres per second
20. The number of real solutions for  $x^2 + 5|x| + 6 = 0$  is
- (a) 0
  - (b) 2
  - (c) 3
  - (d) 4
21.  $\lim_{x \rightarrow \infty} \left(1 - \frac{4}{x-1}\right)^{3x-1}$  is
- (a)  $e^{-12}$
  - (b)  $e^{12}$
  - (c)  $e^{-4}$
  - (d)  $e^{-3}$

22. Let  $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx$  and  $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$ , then which of the following is true?
- (a)  $I < \frac{2}{3}$  and  $J > 2$   
 (b)  $I > \frac{2}{3}$  and  $J < 2$   
 (c)  $I > \frac{2}{3}$  and  $J > 2$   
 (d)  $I < \frac{2}{3}$  and  $J < 2$

23. The sum of the infinite series

$$1 + \frac{1}{4 \times 2!} + \frac{1}{16 \times 4!} + \frac{1}{64 \times 6!} + \dots$$

is

- (a)  $\frac{e-1}{2\sqrt{e}}$   
 (b)  $\frac{e+1}{2\sqrt{e}}$   
 (c)  $\frac{e-1}{\sqrt{e}}$   
 (d)  $\frac{e+1}{\sqrt{e}}$
24. A traffic light runs repeatedly through the following cycle: green for 30 seconds, then yellow for 3 seconds, and then red for 30 seconds. Jack picks a random three - second time interval to watch the light. What is the probability that the color changes while he is watching?
- (a)  $\frac{1}{3}$   
 (b)  $\frac{1}{7}$   
 (c)  $\frac{1}{10}$   
 (d) None of the above

25. The term independent of  $x$  in the binomial expansion of

$$\left( \frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}} \right)^{10}$$

is

- (a) 4  
 (b) 120  
 (c) 210  
 (d) 310

26.  $\lim_{n \rightarrow \infty} \left( \frac{n!}{n^n} \right)^{\frac{1}{n}}$  is equal to

- (a)  $e$
- (b)  $\frac{1}{e}$
- (c)  $\frac{\pi}{4}$
- (d)  $\frac{4}{\pi}$

27. Largest possible area of a right angled triangle having hypotenuse of length 4cm is equal to

- (a) 3 sq cm
- (b) 4 sq cm
- (c) 5 sq cm
- (d) None of the above

28. Let  $f$  be a one-to-one function with domain  $\{x, y, z\}$  and range  $\{1, 2, 3\}$ . It is given that exactly one of the following statements is true and the remaining two are false:  $f(x) = 1$ ,  $f(y) \neq 1$  and  $f(z) \neq 2$ . Then  $f^{-1}(1)$  is equal to

- (a)  $x$
- (b)  $y$
- (c)  $z$
- (d) None of the above

29. If  $f(x) = ae^{2x} + be^x + cx$  satisfies the conditions  $f(0) = 1$ ,  $f'(\log 2) = 31$  and  $\int_0^{\log 4} (f(x) - cx)dx = \frac{39}{2}$ , then

- (a)  $a = 5, b = 6, c = 3$
- (b)  $a = 5, b = -6, c = 3$
- (c)  $a = -5, b = 6, c = 3$
- (d) None of the above

30. If  $x_1, x_2, x_3$  and  $x_4$  are the roots of the equation

$$x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x \cos \beta - \sin \beta = 0$$

then  $\tan^{-1}(x_1) + \tan^{-1}(x_2) - \tan^{-1}(x_3) - \tan^{-1}(x_4)$  is equal to

- (a)  $\beta$
- (b)  $\frac{\pi}{2} - \beta$
- (c)  $\pi - \beta$
- (d) None of the above